

Sexual reproduction as bet-hedging

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Abstract

In evolutionary biology, bet-hedging refers to a strategy that reduces the variance of reproductive success at the cost of reduced mean reproductive success. In unpredictably fluctuating environments, bet-hedgers benefit from higher geometric mean fitness despite having lower arithmetic mean fitness than their specialist competitors. We examine the extent to which sexual reproduction can be considered a type of bet-hedging, by clarifying past arguments, examining parallels and differences to evolutionary games, and by presenting a simple model examining geometric and arithmetic mean payoffs of sexual and asexual reproduction. Sex typically has lower arithmetic mean fitness than asex, while the geometric mean fitness can be higher if sexually produced offspring are not identical. However, asexual individuals that are heterozygotes can gain conservative bet-hedging benefits of similar magnitude while avoiding the costs of sex. This highlights that bet-hedging always has to be specified relative to the payoff structure of relevant competitors. It also makes it unlikely that sex, at least when associated with significant male production, evolves solely based on bet-hedging in the context of frequently and repeatedly occupied environmental states. Future work could usefully consider bet-hedging in open-ended evolutionary scenarios with *de novo* mutations.

Keywords: Bet-hedging, Environmental fluctuation, Evolutionary games, Geometric mean fitness, Sexual reproduction

1 Introduction

Evolutionary dynamics in natural populations are under the combined effect of directional selection and randomness that comes from various sources, including environmental fluctuations and demographic stochasticity. Accurate predictions of evolutionary dynamics depend, in principle, on all the moments of the fitness distribution of individuals and their relative weights. In general, populations tend to be driven towards phenotypes that maximise the odd moments (mean fitness being the first moment) while minimising the even moments of their fitness distributions (variance being the second moment) (Rice, 2008). This implies that the adverse change of one moment can potentially be compensated by the beneficial changes of other moments. Most attention has been placed on the possibility that decreased mean fitness might be sufficiently compensated for by a concomitant decrease of the variance in fitness, such that the strategy with diminished mean fitness outcompetes others over time (Philippi and Seger, 1989). Because strategies that gain success by manipulating fitness variance intuitively fit the idea of “hedging one’s bets” (Starrfelt and Kokko, 2012), this has given rise to a precise biological meaning of the phrase “bet-hedging” (Slatkin, 1974): it refers to strategies that have diminished arithmetic mean fitness, but also reduced variance (and are often studied with the aid of geometric mean fitness).

Bet-hedging bears some similarity to mixed strategies in evolutionary games (the phrase “optimal mixed strategies” (Haccou and Iwasa, 1995, 1998) has been used near-synonymously with bet-hedging under non-game-theoretical contexts): some forms of bet-hedging imply the production of

different kinds of offspring (e.g. different sizes of tubers in the aquatic macrophyte *Scirpus maritimus*, (Charpentier et al., 2012)). Although both bet-hedging and mixed strategies (in game theory) can lead to a mix of phenotypes in the population, there are two important differences between the concepts: first, the adaptive reasoning is different, and second, bet-hedging can also occur without phenotypic variation. To explain the first difference: In evolutionary games, the payoff of an individual depends on the action of other individuals in the population. This is not a requirement in bet-hedging, where the payoff is typically thought to be determined by the stochastically varying environment (though, as our examples show, others' presence can matter too: e.g. sexual reproduction to diversify one's offspring to cope with environmental change would not work if diversity has been lost). A typical context in which bet-hedging is discussed is rainfall that varies over time (Seeger and Brockmann, 1987; Starrfelt and Kokko, 2012). Under such conditions it can then be beneficial if an individual can produce both wet-adapted and dry-adapted offspring, so that regardless of the conditions in a given year, some fraction of offspring will survive; a non-bet-hedger's entire genetic lineage might disappear as soon as an environmental condition occurs to which it is not adapted.

The second difference between mixed strategies and bet-hedging is that the latter can work without there being a "mix" of any kind. Instead of diversifying offspring, a so-called *conservative* way of bet-hedging is to produce only one type of offspring that performs relatively well under all different environments, while not being the best under any of them ("a jack of all trades is the master of none"). This can also reduce fitness variance, and qualify as

50 bet-hedging if it is achieved at the cost of reduced mean fitness.

51 One prominent example that seems to have the characteristics of bet-
 52 hedging, but is less often mentioned in a bet-hedging context, is sexual repro-
 53 duction, where offspring are formed using genetic material from two parents
 54 (because nature is diverse there are definitional complications and grey zones
 55 regarding what counts as sex; see Lehtonen and Kokko (2014)). Producing
 56 offspring in this way, as opposed to the simpler option of asexual reproduc-
 57 tion, incurs costs in many different ways (reviews: Lehtonen et al. (2012);
 58 Meirmans et al. (2012)). The best known cost, and the one we focus on here,
 59 is the two-fold cost of males: if the offspring sex ratio is 1:1 and males and
 60 females are equally costly to produce, a mother will use 50% of her resources
 61 on offspring that do not themselves contribute material resources to the next
 62 generation (Maynard Smith, 1978), and this slows the growth of sexual pop-
 63 ulations compared with asexual ones. Consequently, sexual reproduction –
 64 when it involves producing males – is expected to lead to a reduction of
 65 mean fitness. But on the other hand, through mixing genetic material from
 66 different lineages, sex provides a potent way of producing offspring whose
 67 genomes differ from each other. If some always do well no matter what the
 68 state of the environment, the variance of reproductive fitness can be reduced
 69 compared with an asexual lineage.

70 Given that effects on genetic diversity are central and much discussed in
 71 the sex literature (e.g. Hartfield and Keightley (2012)), it is surprising that
 72 the biological literatures on bet-hedging and on sex are relatively separate.
 73 Mixed strategies have been shown to be advantageous in a fluctuating envi-
 74 ronment (Haccou and Iwasa, 1995, 1998; McNamara et al., 1995). Haccou

75 and Iwasa (1995) have shown that the optimal strategy can involve bet-
 76 hedging under a fluctuating environment in unstructured populations, and
 77 showed how to calculate the strategy explicitly for a given payoff function
 78 and a given distribution of the environmental parameters. In addition, the
 79 optimal bet-hedging strategy is robust against small perturbations of the
 80 distribution of environmental conditions and/or the payoff function (Haccou
 81 and Iwasa, 1998). Cooperative games between kin can also help maximise the
 82 geometric mean fitness of species in fluctuating environments (McNamara,
 83 1995). Furthermore, the strategy that maximises the geometric mean fitness
 84 is more likely to evolve in species of non-overlapping generations compared
 85 to species with substantial parental survival. In the latter case, the strategy
 86 that maximises the arithmetic mean fitness is more likely to evolve (Haccou
 87 and McNamara, 1998). The review of Grafen (1999) discusses different ways
 88 of optimising reproductive fitness in a fluctuating environment. None of these
 89 studies, however, have explicitly pointed out that sexual reproduction can be
 90 a form of bet-hedging.

91 Williams (1975) in his classic book on sex discusses a “lottery model”
 92 using the verbal analogy of buying ever more copies of the same number on a
 93 lottery ticket (asexual reproduction) vs. buying fewer but a more diverse set
 94 of numbers (sexual reproduction). The analogy to a real-life lottery is not
 95 perfect, in the sense that asexually produced offspring are often not totally
 96 redundant copies of each other, i.e. they do not necessarily have to share
 97 the prize if both have a winning number: two asexually produced offspring
 98 usually leave more descendants than just one, especially if they disperse to
 99 different localities and no longer compete for the same resources ((Williams,

100 1975) p.16). The correspondence between Williams' lottery model and bet-
 101 hedging, on the other hand, appears perfect. But Williams (1975) did not use
 102 explicit bet-hedging terminology, probably because it had only very recently
 103 been imported to evolutionary terminology (Slatkin, 1974).

104 Williams (1975) emphasised the need to consider the spatial arrangement
 105 of offspring to determine whether, e.g., 10 "winning tickets" can win 10 prizes,
 106 which requires dispersal to avoid competition with relatives, or are expected
 107 to win less ((Williams, 1975) p.53). The emphasis in Williams' idea is that
 108 the winning numbers vary over time (but not necessarily over space). In a
 109 context where dispersal is limited, a similar idea has been formulated empha-
 110 sising resource diversity rather than its temporal fluctuations. The relevant
 111 metaphor is a "tangled bank", a rather poetic phrase that has its origin in
 112 Darwin's *On the origin of species*. Darwin contemplated "a tangled bank,
 113 clothed with many plants of many kinds, with birds singing on the bushes,
 114 with various insects flitting about, and with worms crawling through the
 115 damp earth..." (Darwin, 1859). Darwin was not talking specifically about sex,
 116 but about life and its evolution in general. Nevertheless, the "tangled bank"
 117 has since acquired a specific meaning (Bell, 1982), becoming a metaphor of
 118 genetic polymorphisms favoured in environments that might not vary much
 119 temporally but that, based on diverse resources present at the same site, offer
 120 multiple niches and the resultant higher total carrying capacity for different
 121 phenotypes as a whole ("the environment is now more fully utilised ..., the
 122 carrying capacity of the diverse population will inevitably exceed that of ei-
 123 ther single clone." (Bell, 1982) p.130). In the "tangled bank" scenario, the
 124 carrying capacity of each single clone depends on the distribution of different

125 niches in the environment. The carrying capacity of the entire diversified
126 population in the heterogeneous environment is larger than any of the single
127 clones.

128 Although the “tangled bank” does not require a temporally fluctuating
129 environment, the diversity of different clones is maintained better if the envi-
130 ronment changes frequently (Bell, 1982). In addition, in the “tangled bank”,
131 the fitness of a single clone depends not only on the abundance of different
132 niches, but also is frequency-dependent when competing for the same niche
133 or invading a new niche (Bell, 1982). Therefore, the “tangled bank” may cap-
134 ture aspects of the benefits of sexual reproduction, but it does not perfectly
135 correspond to bet-hedging.

136 2 Bet-hedging via heterozygotes and sexual re- 137 production

138 We examine in the following the conditions under which sexual reproduc-
139 tion might spread as a form of bet-hedging. Our model considers a large
140 well-mixed population where a proportion s of the young produced are male.
141 Note that our assumption of large (infinite) population size allows us to focus
142 on the effects of environmental stochasticity without confounding effects of
143 demographic stochasticity. Asexual individuals are all female. The adapta-
144 tion to the amount of rainfall in the environment is determined by a diploid
145 genetic locus that has two alleles. The AA genotype is well adapted to the
146 wet environment, whereas the aa genotype is dry-adapted. The heterozy-

147 gote Aa has intermediate fitness in both environments, but not necessarily
 148 exactly the mean of aa and AA. Example fitness values for each genotype
 149 under different environments are show in matrix (1).

$$\begin{matrix} & \text{Wet} & \text{Dry} \\ \begin{matrix} AA \\ Aa \\ aa \end{matrix} & \begin{pmatrix} 8 & 2 \\ 4.5 & 4.5 \\ 2 & 8 \end{pmatrix} \end{matrix} \quad (1)$$

150 Consider a case where wet and dry environments occur at equal frequen-
 151 cies, and all individuals are asexual females. Table 1 shows the arithmetic
 152 mean and geometric mean fitness of the different asexual types. The het-
 153 erozygote (Aa) has the lowest arithmetic mean fitness, but the highest ge-
 154 ometric mean fitness, which predicts higher evolutionary success if we ig-
 155 nore higher moments of the fitness distribution (Starrfelt and Kokko, 2012).
 156 The asexual heterozygotic form becomes thus a bet-hedging strategy when
 157 compared with the two other asexual homozygotic forms. This form of bet-
 158 hedging is *conservative*: all Aa individuals have the same expected fitness
 159 under both environmental conditions.

Table 1: The payoff structure under wet and dry years: the arithmetic mean (AMean) and the geometric mean (GMean) of the payoffs of asexual lineages, as well as of a sexual population assumed to be at the Hardy-Weinberg equilibrium.

	Wet	Dry	AMean	GMean
asex-AA	8	2	5	4
asex-Aa	4.5	4.5	4.5	4.5
asex-aa	2	8	5	4
sex-population	4.75(1-s)	4.75(1-s)	4.75(1-s)	4.75(1-s)

160 In contrast to the conservative approach of the asexual heterozygotes, the
 161 sexual population as a whole can also be seen to bet-hedge, in this case by
 162 producing offspring of different genotypes. It is therefore of interest to ask if
 163 sex is a bet-hedger with respect to AA, Aa, aa or perhaps all of them. The
 164 comparison is more complicated than the above one, not only because sex
 165 produces young that differ from each other (and thus differ in the long-term
 166 growth rate impacting the original parent's contribution to the future gene
 167 pool), but also because the frequencies of genotypes in the offspring of any
 168 given parent depend on the genetic composition of the population as a whole
 169 – which in turn depends on how selection has worked on it in the recent past:
 170 a run of wet years will have favoured the A allele, dry years do the opposite.

171 We initially assume that the sexual population is always under Hardy-
 172 Weinberg equilibrium (Hardy, 1908; Weinberg, 1908) and that the two alleles
 173 are equally abundant. This is a strong assumption that is expected to be
 174 violated as soon as selection is applied, but we nevertheless consider it as
 175 a useful thought experiment, because the genetic background that an allele
 176 faces is then constant across generations (genotypic proportions are always
 177 expected to be $x_{AA} = 1/4$, $x_{Aa} = 1/2$, and $x_{aa} = 1/4$). Given that only
 178 females contribute directly to offspring production (males only impact the
 179 genetic diversity of young she produces), the expected growth rate of the
 180 sexual population equals $(8/4 + 4.5/2 + 2/4)(1 - s) = 4.75(1 - s)$, where s is
 181 the proportion of males. If the sexual population achieves this growth rate
 182 in every year (which requires that it maintains itself at the Hardy-Weinberg
 183 equilibrium), and as long as s is not too large, it has performed perfect bet-
 184 hedging as the geometric mean now equals the arithmetic mean, which is its

185 maximum value.

186 But is this geometric mean fitness higher than that of the specialist asex-
187 uals (AA and aa)? The answer depends on the cost of sex, which we here
188 model as the proportion s of offspring developing as males. Sex beats AA
189 or aa asexual genotypes if $s < 0.158$, while beating the bet-hedging asexual
190 genotype (Aa) is harder: it only occurs if $s < 0.0526$.

191 While the example shows that sexual reproduction can, in principle, be
192 a bet-hedging strategy, it simultaneously shows how difficult it is for sex to
193 evolve based on this benefit alone, especially if competing against asexual
194 types that also bet-hedge (conservatively). The cost of males is captured by
195 s , and the more females produce sons, the higher this cost. Why males exist is
196 a separate evolutionary conundrum from why sex exists: the alternative that
197 is relevant for the “why males?” question is still sex, but without having some
198 individuals specialise in the male strategy that fails to contribute directly to
199 population growth. This question has its own set of game-theoretical answers
200 (Bulmer and Parker, 2002; Lessells et al., 2009; Lehtonen and Kokko, 2011);
201 the short summary is that (1) males can invade sexual populations despite
202 the reduced growth rate, (2) their existence increases the vulnerability of
203 sexual populations to invasion by asexuals, (3) if a population only consists
204 of (sexual) females and males, sex ratios evolve to $s = 0.5$ under quite general
205 conditions (West, 2009).

206 In Table 1, the arithmetic mean decreases rapidly with an increasing pro-
207 duction of males, and any primary sex ratio greater than 15.8% males leads
208 to sexuals being unable to resist invasion by any of the asexual options. Be-
209 cause male presence typically leads to much higher sex ratios, sex is unlikely

210 to persist due to its bet-hedging benefits alone, at least in the simplistic
211 setting of Table 1.

212 Sexual populations can resist invasions somewhat better (i.e. up to a
213 larger fraction of sons produced) if the dimensionality of bet-hedging in-
214 creases (i.e. it involves multiple traits). For example, besides the A/a locus
215 that determines an individual's fitness in response to the amount of rain-
216 fall, consider another diploid locus that impacts the adaptedness to high or
217 low temperatures. Assume that an individual of the BB genotype is hot-
218 adapted, an individual of the bb type is cold-adapted, and the Bb genotype
219 is intermediate. Also assume the payoff matrices for rainfall and temperature
220 adaptation has the same structure:

$$\begin{array}{cc} & \begin{array}{cc} \text{Wet} & \text{Dry} \end{array} \\ \begin{array}{c} AA \\ Aa \\ aa \end{array} & \begin{pmatrix} 8 & 2 \\ 4.5 & 4.5 \\ 2 & 8 \end{pmatrix} \end{array} \quad \begin{array}{cc} & \begin{array}{cc} \text{Hot} & \text{Cold} \end{array} \\ \begin{array}{c} BB \\ Bb \\ bb \end{array} & \begin{pmatrix} 8 & 2 \\ 4.5 & 4.5 \\ 2 & 8 \end{pmatrix} \end{array}. \quad (2)$$

221 If different traits interact multiplicatively to determine the final fitness,
222 then an AABB individual has payoff of 64 if the environment is both wet
223 and hot (WH), 16 if the environment is wet but cold (WC), or dry but hot
224 (DH), and 4 if the environment is both dry and cold (DC). Table 2 gives the
225 complete list of payoffs of different genotypes under different environments.

226 For simplicity we may assume that the four environmental conditions
227 occur at equal probabilities (i.e., rainfall does not make the year cooler or
228 vice versa). If we once again assume Hardy-Weinberg equilibrium and equal

Table 2: Payoff of different genotypes under four different environmental conditions, when there are two traits impacting fitness.

genotype	freq.	WH	WC	DH	DC
AABB	1/16	64	16	16	4
AABb	1/8	32	32	8	8
AAbb	1/16	16	64	4	16
AaBB	1/8	32	8	32	8
AaBb	1/4	16	16	16	16
Aabb	1/8	8	32	8	32
aaBB	1/16	16	4	64	16
aaBb	1/8	8	8	32	32
aabb	1/16	4	16	16	64

allele frequencies, the sexual population achieves a growth rate $22.5625(1 - s)$ in every environmental setting, which also implies a geometric mean of $22.5625(1 - s)$. The geometric mean for the asexuals is 16 for homozygote specialists (AABB, AAbb, aaBB, aabb), 18 for those who bet-hedge conservatively with respect to one trait only (AABb, aaBb, AaBB, Aabb), and 20.25 for the asexual genotype that conservatively hedges its bets with respect to both traits (AaBb). The sexual population can beat any asexual genotype if $s < 0.1025$, it can be beaten by the best bet-hedging asexual AaBb but not by others if $0.1025 \leq s < 0.2022$, it can be beaten by all bet-hedging asexuals (AABb, aaBb, AaBB, Aabb and AaBb) but beat the full homozygotes if $0.2022 \leq s < 0.2909$, and remains vulnerable to invasion by any asexual type if s exceeds 0.2909.

We used specific numerical values in the example above, which raises the question how these generalise to other scenarios of allelic dominance, including dominance-recessive, heterosis and inbreeding depression. It has been shown that sexual population can reach all possible phenotypic states

if and only if the hereditary system is either dominant-recessive or maternal or the combination of these (Garay and Garay, 1998). We show in appendix A that under the hereditary scheme where one allele is completely dominant over the other allele, the sexual heterozygote ceases to be a bet-hedging strategy since both its arithmetic mean and geometric mean fitness become equal to those of the asexual homozygote. Stronger dominance, on the other hand, improves the geometric mean fitness of the sexual population, making it potentially easier to outcompete asexuals.

3 Numerical simulations

In the previous section, we used the frequency distribution of different genotypes at Hardy-Weinberg equilibrium for calculating the arithmetic and geometric mean payoff of the sexual population. This is convenient, as it allows us to examine the situation as if the sexual population reached the same growth rate in every environmental setting (it makes sex achieve perfect bet-hedging in the sense that the geometric mean payoff equals the arithmetic mean payoff). However, in reality sex will fail to achieve this perfection, because the genetic environment encountered by a sexual population will be a function of past selection. There will then also be temporal variation in the distributions of genotypes, and sex is likely to fail to achieve perfect bet-hedging. The geometric mean fitness will then drop below the arithmetic mean fitness.

Since the pioneering work of Maynard Smith (Maynard Smith, 1971, 1976), Hamilton (Hamilton et al., 1981) and Bell (Bell, 1982), it has been

known that the rate of temporal fluctuations can matter for the evolution of sex. In our setting above, the frequency of switches between wet and dry environments determines how far from equilibrium genotype frequencies will deviate over time. In the following we therefore use numerical simulations to show a more realistic picture of the competition dynamics between sexual and asexual populations.

3.1 Environmental fluctuations

Here we relax the assumption of Hardy-Weinberg equilibrium: it is only used as a starting state for sexual reproduction, and the following dynamics are computed according to a realised run of fluctuations of the environmental state. Assume that the wet and dry environments follow each other in a manner that can be captured by discrete-time Markov chains (i.e. the transition probability from one state to another does not depend on how long the environment has spent in the current state). The transition probabilities between states can be written in the matrix form

$$\begin{array}{cc} & \begin{array}{cc} \text{Wet} & \text{Dry} \end{array} \\ \begin{array}{c} \text{Wet} \\ \text{Dry} \end{array} & \begin{pmatrix} 1 - p_{wd} & p_{dw} \\ p_{wd} & 1 - p_{dw} \end{pmatrix}, \end{array} \quad (3)$$

in which p_{wd} denotes the probability that the environment changes from wet to dry in a year, and p_{dw} is the probability that the environment changes from dry to wet in a year. The normalized dominant right eigenvector represents the stationary distribution of the environmental states (Caswell, 2001),

and has the value $(p_{wd} / (p_{wd} + p_{dw}); p_{dw} / (p_{wd} + p_{dw}))$. The subdominant eigenvalue $\rho = 1 - p_{wd} - p_{dw}$ in turn corresponds to the correlation between the environmental states at times t and $t + 1$ (Caswell, 2001). Therefore, consecutive environmental states are negatively autocorrelated if $\rho < 0$, positively autocorrelated if $\rho > 0$, and uncorrelated if $\rho = 0$. In the extreme case where $p_{wd} = p_{dw} = 1$, we have $\rho = -1$ and wet and dry environments alternate, whereas in the other extreme case where $p_{wd} = p_{dw} = 0$, we have $\rho = 1$ and the environment stays in the initial state forever.

3.2 Simulation results

To focus on the effect of environmental fluctuations, we exclude the effect of demographic stochasticity and drift by assuming that the population size is very large. We use the fixation probability of the invading type as a proxy for the relative advantages of different types. We do this by setting up a population consisting of an initial proportion 0.02 of the invading type, competing against one of the three possible alternative types. We assume that, for sexuals, the growth rate is proportional to $1 - s$ (the frequency of females), and the proportion of AA, Aa and aa young are derived by assuming that both male siring propensity and the female propensity to reproduce are proportional to that genotype's payoffs (this covers at least two possible biological interpretations: survival probabilities are proportional to payoffs and thereafter mating is random, with each mating producing an equal number of offspring; or that the fecundity of females, as well as the siring success of males, is proportional to payoffs. As a caveat, note that the two cases can

310 be mapped to each other directly only in unstructured populations. If the
 311 population has overlapping generations, selecting on survival and reproduc-
 312 tion have to be treated separately from each other (Haccou and McNamara,
 313 1998; Li et al., 2016)).

314 The invasion is tracked until one of three mutually exclusive events have
 315 happened: (a) the invading type has reached frequency 0.9999 or higher (we
 316 consider this a successful invasion, and fixation is reached), (b) the invading
 317 type's frequency falls below 0.0001 (we assume that the invasion failed), or
 318 (c) neither (a) nor (b) have happened by generation 10^6 (we consider this a
 319 coexistence scenario, but in practice event (c) never happened). The Octave
 320 codes for all numerical simulations are provided in Supplementary Informa-
 321 tion. The sexual population starts from the Hardy-Weinberg equilibrium
 322 state, with a proportion of 0.25 AA, 0.5 Aa and 0.25 aa types. The payoff of
 323 each genotype under different environments follows matrix (1), and fixation
 324 probabilities are estimated from 10^4 independent realisations. Because the
 325 payoffs of the asexual AA and aa types are symmetric, and the wet and dry
 326 environments occur at equal frequencies, they have identical fixation proba-
 327 bilities when invading or being invaded by a sexual population. Therefore,
 328 without loss of generality, we use the asexual AA to represent the case of
 329 asexual homozygotes in Figure 1.

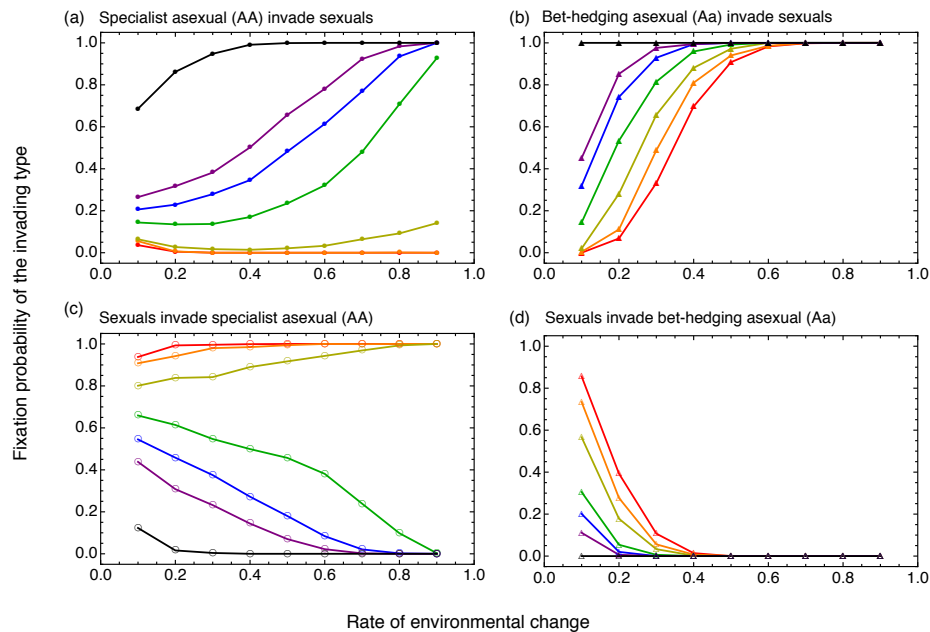


Figure 1: Fixation probability of the invading types under various rates of environmental change for populations following payoff matrix (1). The x-axis represents the rate of environmental change, assuming $p_{wd} = p_{dw}$. Colours from red to purple to black represent sexual population of different sex ratios (0.01, 0.02, 0.04, 0.08, 0.12, 0.16, and 0.5). The larger the sex ratio, the higher the cost of sex. These figures are based on 10^4 realisations per parameter value, and never required stopping the simulation at generation 10^6 (i.e. either fixation is reached or the invader went extinct).

330 The figure confirms that sex has a difficult time invading asexual strate-
331 gies if $s = 0.5$. If we elevate the chances for sexual reproduction to invade
332 others by allowing $s < 0.5$, then cases where sex outcompetes specialist asex-
333 uals (AA or aa) still typically do not predict that sex can also outcompete
334 bet-hedging asexuals (comparing the left and right panels: curves are almost
335 invariably higher on the right than on the left when considering an asexual
336 invasion, and are always lower on the right than on the left when considering
337 a sexual invasion). Whether fast or slow environmental fluctuations are best

for sex is surprisingly complex. At very small s , sexuals are more likely to invade asexual homozygotes (and also resist their invasion attempts) if the environment changes fast. Other values of s predict the opposite. This complexity contrasts with early work on geometric mean fitness in the context of sex (Hamilton et al., 1981), predicting that a fast changing environment is beneficial to the maintenance of sex in general. But there are crucial differences between the payoff structures in his model and ours. (Note that Hamilton did not call Hamilton's temporal fluctuation model bet-hedging).

The success of invasion is likely to depend on how long allelic diversity persists in the population. If the payoff of the heterozygote is low, and the environment changes relatively slowly, genetic diversity might become extinguished even before the asexual mutant is introduced. When the sexual population exists alone, it is possible that one allele, either a or A , is lost (examples: Figure 2a-b, mean time to extinction: Figure 2c-d). The better the heterozygote (Aa) payoff (Figure 2c), and the faster the environmental fluctuations (Figure 2d), the longer the coexistence time of both alleles. If one allele has already been lost, sex behaves genetically like an asexual homozygote (losing its bet-hedging benefit), but still paying the cost of sex. Note that a population that bet-hedges via asexuality (Aa) does not suffer from this risk, as both alleles are kept intact in this lineage in every generation. In this sense, conservative bet-hedging represented by asexuality may perform better than the diversified bet-hedging represented by sexual reproduction.

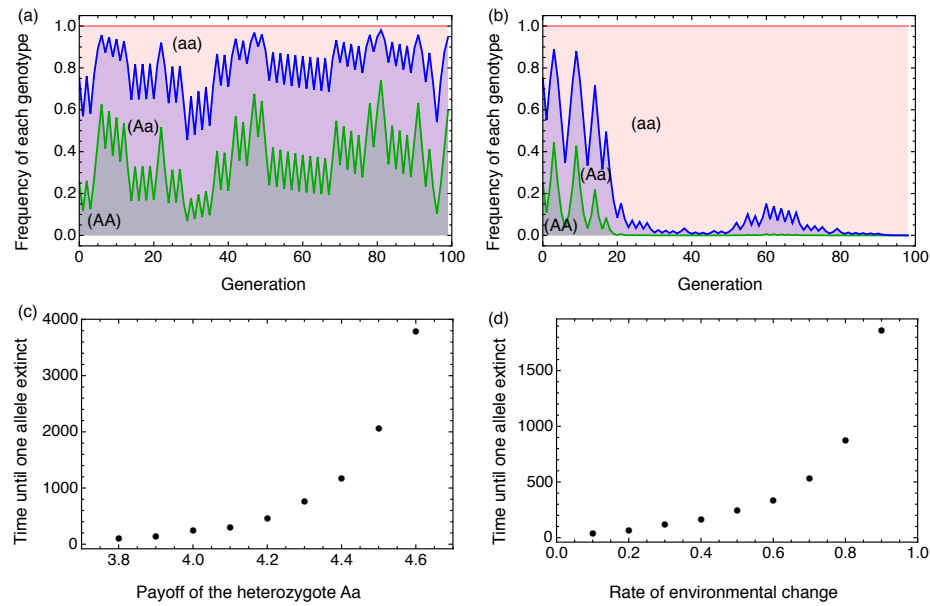


Figure 2: Examples of genetic diversity in a purely sexual population (no mutation to asexuality), where diversity is maintained (panel a) or lost (panel b) under environmental fluctuations that are tracked for 100 generations. The two trajectories are from simulations with identical parameter settings. In both cases, the rate of environmental change $p_{wd} = p_{dw} = 0.75$, and the payoff of the heterozygote is set to 3.8 under both environmental conditions. The vertical height of regions of various colours represent the frequencies of different genotypes. (c) The mean time to the disappearance of one allele as a function of varying heterozygote payoffs when $p_{wd} = p_{dw} = 0.5$, and (d) the mean time to the disappearance of one allele as a function of the rate of environmental change when $p_{wd} = p_{dw}$ and heterozygote payoff is 4.0 under both environmental conditions. In all simulations, the payoffs of the homozygotes follow payoff matrix (1). In panels (c) and (d), one allele is considered to have gone extinct if the frequencies of both the corresponding homozygote and the heterozygote are smaller than 10^{-4} .

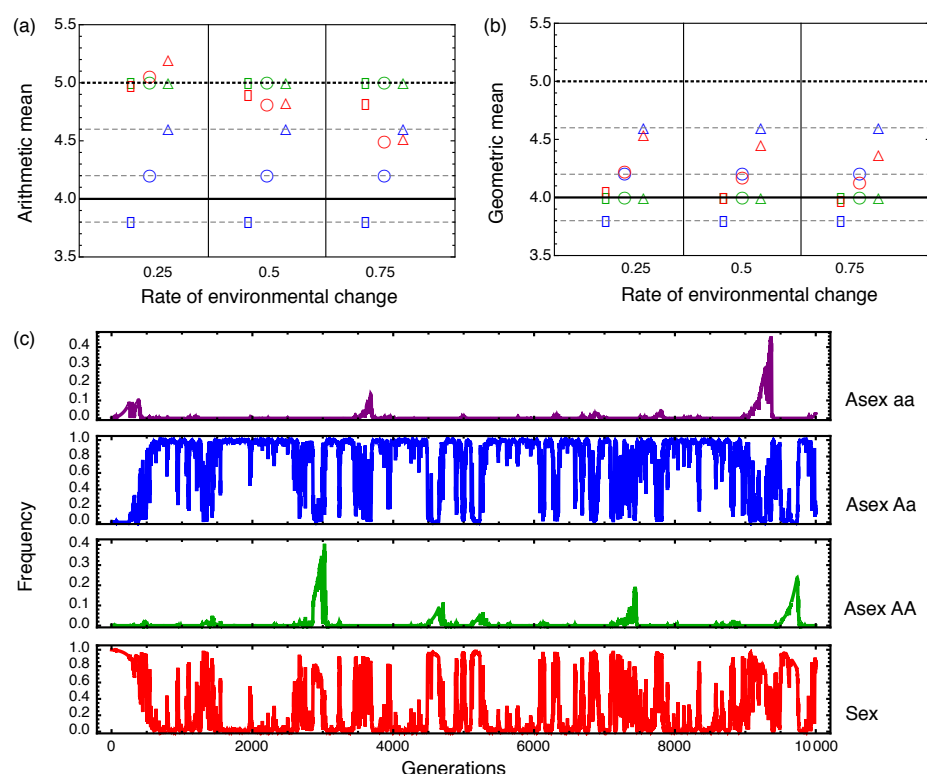


Figure 3: (a) Arithmetic mean payoffs and (b) geometric mean payoffs of the asexual homozygote (green), asexual heterozygote (blue) and the sexual population (red), computed over 500 generations when the payoffs of the asexual homozygotes follow matrix (1) and the sex ratio of the sexual population is set to $s = 0.01$. Symbols of different shapes represent different payoffs of the heterozygote: square, circle and triangle stand for 3.8, 4.2, and 4.6 respectively. The black dotted line is the expected arithmetic mean payoff of the asexual homozygotes, the black solid line is the expected geometric mean payoff of the asexual homozygotes, and the grey dashed lines are the expected arithmetic and geometric mean payoff of the asexual heterozygote. (c) Frequency dynamics of the sexual population and each asexual genotype population under a changing environment over 10000 generations. In each panel, the x-axis is time (the elapsed number of generations), and the y-axis is the frequency of each type. All four panels are from the same instance of simulation. The heterozygote payoff is set to 4.2, and the rate of environmental change is $p_{wd} = p_{dw} = 0.5$. The simulation starts with a pure sexual population with 0.25 AA, 0.5 Aa and 0.25 aa genotypes, but each individual may mutate to being asexual if previously sexual, or sexual if previously asexual, at rate 0.0001 per generation.

360 A key finding is therefore that sex cannot easily outcompete asexual forms
 361 based on bet-hedging benefits alone (Figure 3). Sex as bet-hedging requires
 362 conditions under which the red symbols are below the dotted line in Figure
 363 3a, and above the solid line in Figure 3b. Only four out of the nine cases
 364 satisfy the requirements (heterozygote payoff 4.2 or 4.6 in combinations with
 365 rate of environmental change 0.5 or 0.75). However, it is possible to construct
 366 cases where sex wins in terms of arithmetic mean fitness but loses in terms
 367 of geometric mean to the conservative asexual bet-hedger (Figure 3c, where
 368 the heterozygote payoff is set to 4.2, and the rate of environmental change is
 369 set to 0.5).

370 4 Discussion

371 There are interesting parallels between sex and bet-hedging theory. Intu-
 372 itively, the costs of sex reduce the fitness of sexual lineages in every gen-
 373 eration that undergoes a sexual life cycle (hence the arithmetic fitness is
 374 reduced), but by diversifying the genotypes of offspring, sex can reduce the
 375 variance in success: in any given year some offspring will survive, while an
 376 asexual specialist proverbially puts “all its eggs in one basket” – leading to
 377 very low success if the year features a mismatch between offspring genotype
 378 and the state of the environment. However, for this to favour sex over asex,
 379 the geometric mean fitness of the former should be elevated above the latter.
 380 Although variance reductions have a beneficial effect on geometric mean fit-
 381 ness, arithmetic mean fitness (which is low for sexual types) simultaneously
 382 sets an upper limit for it, and hence it is not easy for sex to reach such high

bet-hedging benefits that its geometric mean fitness is the best of all competing strategies. In other words, the fact that sexual reproduction shows features of bet-hedging is not the same statement as the claim that bet-hedging provides strong enough benefits for the evolution and maintenance of sex. This is especially true since sex may have to compete against another type of bet-hedger: that of asexual heterozygotes, which avoid paying the cost of sex but may also achieve bet-hedging if their genotype performs reasonably well under all considered environmental conditions. This highlights that (a) it is important to specify that a strategy is performing bet-hedging relative to another strategy, and be explicit about the identity of the relevant competitor, and (b) that it would be premature to consider bet-hedging as a major driving force behind the maintenance of sex, at least under the simplifying assumptions of the current model.

Fast and unpredictable changes of the environment have been found to favour bet-hedging (Haccou and Iwasa, 1995) and facilitate the maintenance of sexual reproduction (Maynard Smith, 1971, 1976; Treisman, 1976; Hamilton et al., 1981; Bell, 1982; Waxman and Peck, 1999; Barbuti et al., 2012), but these authors did not use bet-hedging terminology. Our model shares a similar genetic structure to Hamilton et al. (1981), but the payoff structures are different. In our model, the two asexual homozygotes are specialists that adapt to different environmental conditions, and the heterozygote has intermediate payoff under both environmental conditions (this makes it a conservative bet-hedger). In Hamilton's model, the homozygotes receive identical payoffs (that depend on environmental conditions), whereas the payoff of the heterozygote is the reciprocal of this payoff. The heterozygote and ho-

408 mozygotes in the model of Hamilton et al. (1981) thus do not correspond to
409 a bet-hedger and two specialists, and therefore, although the model shows
410 that sex is beneficial under a fast changing environment, it did not aim to
411 capture the evolutionary dynamics under the bet-hedging context.

412 Compared to classic bet-hedging scenarios where the bet-hedger always
413 has the same payoff under the same environment (Starrfelt and Kokko, 2012),
414 sexual reproduction as bet-hedging brings in additional features. In the sex-
415 ual population, the arithmetic mean payoff in each generation is determined
416 not only by the environment, but also the frequency distribution of all geno-
417 types, the sex ratio, and possibly other costs or benefits from sexual repro-
418 duction. In addition, if mutations between sexual and asexual populations
419 are allowed, more than one type of bet-hedging strategy can (at least tem-
420 porarily) coexist, and it is insightful to remember that there can be asexual
421 heterozygotes that bet-hedge conservatively, as opposed to the diversified
422 bet-hedging of the sexual population.

423 Both theoretical and experimental work on the evolution of sex show
424 complications that highlight the simplicity of any two-environment model
425 (indeed, in our model too, increasing the dimensionality of the system helps
426 maintain sex). We have followed a tradition in bet-hedging theory where
427 2 (or 4) types of environment can be adapted to with one (or two) traits.
428 Modern research on genetic variation reveals that there is surprisingly much
429 polygenic variation present in populations (Charlesworth, 2015), and fitness
430 landscapes are often complex. Recent research on sex has revealed the po-
431 tential importance of processes such as clonal interference (McDonald et al.,
432 2016; Sharp and Otto, 2016), which tends to erode the success of asexual

433 lineages over time because they are slow to acquire multiple novel mutations
434 that aid adaptation. Sex improves the rate with which innovations end up
435 in the same organism, while asexual lineages tend to fail in having access
436 to the most “up to date” genetic background, especially if the environment
437 keeps changing. The detrimental interference between competing clones that
438 have acquired one or another beneficial allele (at different loci) eventually
439 makes asexuality an inferior competitor in the adaptive race. While this
440 is a very different situation from what bet-hedging theory traditionally has
441 considered, there is scope to fill this gap: the gist of the argument is that
442 the asexual lineages experience diminishing geometric fitness once timescales
443 become long enough that novel beneficial mutations begin playing a role. Sex
444 and the diversity it creates can help diversify the genetic backgrounds where
445 new mutations can be selected for.

446 Among the classic literatures, the payoff structure in Treisman (1976) is
447 the closest to ours, and it also captures some of the above ideas about the
448 environment changing to something never experienced before. In Treisman
449 (1976), different alleles interact additively and give the diploid individual
450 a phenotype (in his words, a “genotypical score”) that impacts female fer-
451 tility but not male siring success. Alleles have effects of -0.5 or 0.5, so
452 that homozygotes have phenotypes -1 or 1, and the heterozygote has an
453 intermediate phenotype of 0. Females (both sexual and asexual) can only
454 breed if their phenotype matches, within tolerable range, the environmental
455 conditions (such as temperature). If the environment keeps changing (e.g.,
456 increasing temperatures), asexual genotypes cannot keep pace with sexuals
457 that produce diversified offspring through recombination; asexual extinction

458 can then follow. Treisman (1976), like the authors mentioned above, did not
 459 use the terminology of bet-hedging, and hence did not analyse the arithmetic
 460 and geometric mean fitness of each genotype.

461 Given that there is both old and new work on sex that could gain con-
 462 ceptual clarity if researchers routinely reported how the winning strategy
 463 (sexual or asexual) performed in terms of arithmetic and geometric mean
 464 fitness, we welcome more work in the areas linking sex and bet-hedging.
 465 Bet-hedging theory has brought about increased understanding of other evo-
 466 lutionary questions from dispersal evolution (Armsworth and Roughgarden,
 467 2005) and dormancy timing (Ellner, 1985; Evans and Dennehy, 2005; Fur-
 468 ness et al., 2015) to antibiotic resistance (Arnoldini et al., 2014), microbial
 469 population dynamics (de Jong et al., 2011) and phenotypic switching (Carja
 470 et al., 2014). It would appear timely to add sexual reproduction to this list.
 471 Even if sex in simplistic settings (like ours) does not reach the status of a
 472 strategy with the highest geometric mean fitness, a bet-hedging perspective
 473 can shed light on the precise reasons why it failed. An interesting question
 474 would be to use this type of analysis to examine cases where sex, e.g. in
 475 situations involving clonal interference and *de novo* mutations, succeeds to
 476 maintain itself against asexual competitors.

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482 International Symposium on Dynamic Games and Applications.

483 A Sex as bet-hedging when one allele domi- 484 nates the other

485 Assume that the A allele fully dominates the a allele. The fitness values of
486 each genotype under different environments are show in matrix (4).

$$\begin{array}{cc} & \text{Wet} & \text{Dry} \\ \begin{array}{c} AA \\ Aa \\ aa \end{array} & \left(\begin{array}{cc} 8 & 2 \\ 8 & 2 \\ 2 & 8 \end{array} \right) \end{array} \quad (4)$$

487 In this case, the payoff of each asexual type and the sexual population is
488 shown in table 3.

Table 3: The payoff structure under wet and dry years when the A allele fully dominates the a allele: the arithmetic mean (AMean) and the geometric mean (GMean) of the payoffs of asexual lineages, as well as of a sexual population assumed to be at the Hardy-Weinberg equilibrium.

	Wet	Dry	AMean	GMean
asex- AA	8	2	5	4
asex- Aa	8	2	5	4
asex- aa	2	8	5	4
sex-population	$6.50(1-s)$	$3.50(1-s)$	$5(1-s)$	$4.77(1-s)$

489 The first observation is that the asexual heterozygote is no longer a bet-
490 hedging strategy, since its payoffs under different environmental conditions

491 become identical to the homozygote AA , and thus its geometric and arith-
 492 metic payoffs no longer fit the requirements of bet-hedging. Under Hardy-
 493 Weinberg equilibrium, the sexual population would have higher geometric
 494 mean payoff and lower arithmetic mean payoff than each asexual type when
 495 $0 < s < 0.162$. This range is larger than that under the case of intermediate
 496 inheritance, where the sexual population beats any asexual homozygote if
 497 $0 < s < 0.158$, and beats the asexual heterozygote if $0 < s < 0.053$.

498 Similar results hold when populations hedge their bets on multiple traits.
 499 Using the case in matrix (2) as an example, if the A allele fully dominates
 500 the a allele, and the B allele fully dominates the b allele, the payoff matrices
 501 for rainfall and temperature adaptation has the following structure:

$$\begin{array}{cc} & \begin{array}{cc} \text{Wet} & \text{Dry} \end{array} \\ \begin{array}{c} AA \\ Aa \\ aa \end{array} & \begin{pmatrix} 8 & 2 \\ 8 & 2 \\ 2 & 8 \end{pmatrix} \end{array} \quad \begin{array}{cc} & \begin{array}{cc} \text{Hot} & \text{Cold} \end{array} \\ \begin{array}{c} BB \\ Bb \\ bb \end{array} & \begin{pmatrix} 8 & 2 \\ 8 & 2 \\ 2 & 8 \end{pmatrix} \end{array}. \quad (5)$$

502 Again, we assume that different traits interact multiplicatively to deter-
 503 mine the final fitness, and the sexual population is under Hardy-Weinberg
 504 equilibrium. Table 4 gives the complete list of payoffs of different genotypes
 505 under different environments.

506 In this case the sexual population has a fitness of $42.25(1 - s)$ under the
 507 WH environment, $12.25(1 - s)$ under the DC environment, and $22.75(1 - s)$
 508 under both WC and DH environments. Therefore, if four different environ-
 509 ments occur at equal frequencies, the arithmetic mean payoff of the sexual

Table 4: Payoff of different genotypes under four different environmental conditions under the dominance hereditary system, when two traits determine the fitness together.

genotype	freq.	WH	WC	DH	DC
AABB	1/16	64	16	16	4
AABb	1/8	64	16	16	4
AAbb	1/16	16	64	4	16
AaBB	1/8	64	16	16	4
AaBb	1/4	64	16	16	4
Aabb	1/8	16	64	4	16
aaBB	1/16	16	4	64	16
aaBb	1/8	16	4	64	16
aabb	1/16	4	16	16	64

510 population is $25(1 - s)$, and the geometric mean fitness is $22.75(1 - s)$. The
511 geometric mean for the asexuals is 16 for all asexual types. In this way, the
512 sexual population beats any asexual population if $0 < s < 0.297$. This range
513 is also larger than the condition ($0 < s < 0.102$) for beating any asexual
514 genotype under the intermediate heredity.

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517 random movement on population dynamics and biodiversity patterns. *The*
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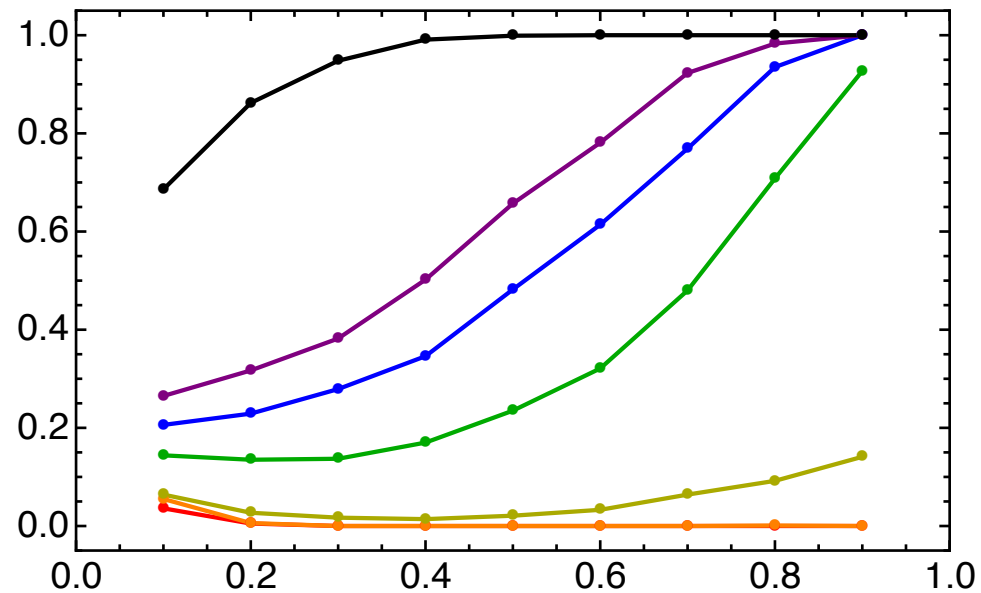
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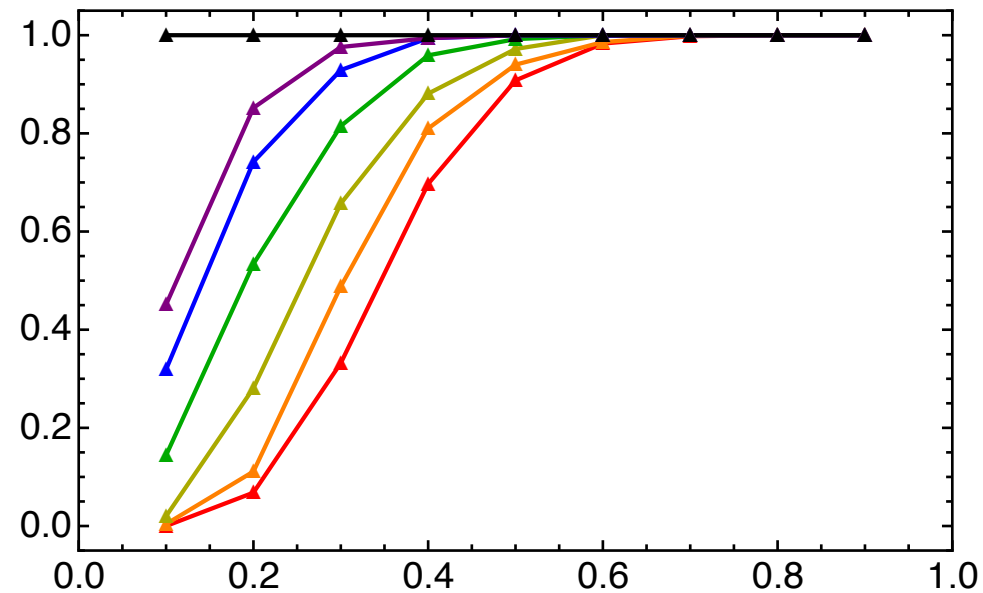
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Fixation probability of the invading type

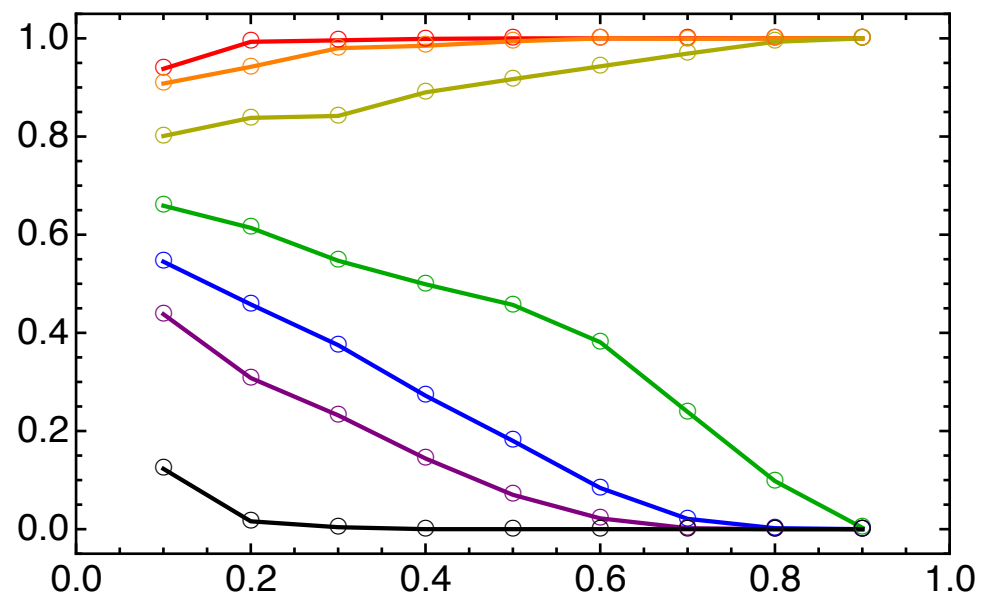
(a) Specialist asexual (AA) invade sexuals



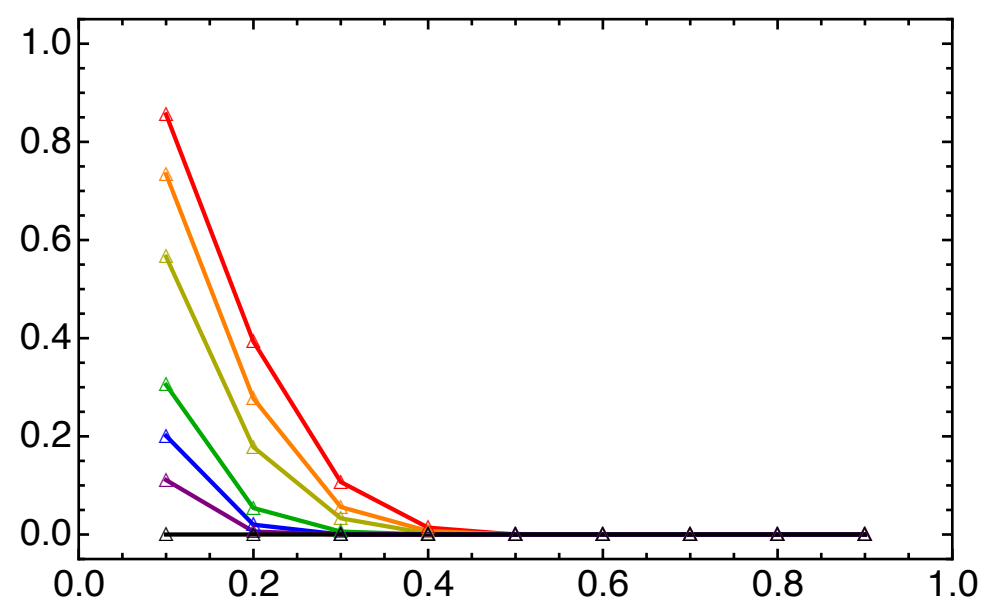
(b) Bet-hedging asexual (Aa) invade sexuals



(c) Sexuals invade specialist asexual (AA)



(d) Sexuals invade bet-hedging asexual (Aa)



Rate of environmental change

